

Appendix A

Mathematics Cognitive Domains Framework: TIMSS 2003 Developmental Project Fourth and Eighth Grades

To respond correctly to TIMSS test items, students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. The first domain, *knowing facts, procedures, and concepts*, covers what the student needs to know, while the second, *applying knowledge and conceptual understanding*, focuses on the ability of the student to apply what he or she knows to solve problems or answer questions. The third domain, *reasoning*, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems.

Knowing Facts, Procedures, and Concepts

Facility in using mathematics, or reasoning about mathematical situations, depends on mathematical knowledge and familiarity with mathematical concepts. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she has understood, the greater the potential for engaging a wide range of problem-solving situations and for developing mathematical understanding.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. *Facts* encompass the factual knowledge that provides the basic language of mathematics, and the essential mathematical facts and properties that form the foundation for mathematical thought.

Procedures form a bridge between more basic knowledge and the use of mathematics for solving routine problems, especially those encountered by many people in their daily lives. In essence a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Knowledge of *concepts* enables students to make connections between elements of knowledge that, at best, would otherwise be retained as isolated facts. It allows them to make extensions beyond their existing knowledge, judge the validity of mathematical statements and methods, and create mathematical representations.

This cognitive domain covers the following behaviors:

Recall	Recall definitions; terminology; number properties; geometric properties; and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).
Recognize	Recognize mathematical objects, shapes, numbers and expressions. Recognize mathematical entities that are mathematically equivalent, e.g. areas of parts of figures to represent fractions, equivalent familiar fractions, decimals and percents; simple algebraic expressions that represent a straightforward situation (eighth grade); different orientations of simple geometric figures; and the nets of simple geometric figures (eighth grade).

Compute	Carry out algorithmic procedures for $+$, $-$, \times , \div , or a combination of these with whole numbers, fractions, decimals and integers. Approximate numbers to estimate computations. Carry out routine algebraic procedures.
Retrieve	Retrieve information from graphs, tables or other sources; read simple scales.
Measure	Use measuring instruments to draw lines, angles, and shapes to given specifications; use units of measurement appropriately; and estimate measures.
Know	Know concepts (e.g., place value; rounding; that length, area and volume are conserved under certain conditions; equal and unequal chance).
Classify/Order	Classify/group objects, shapes, numbers and expressions according to common properties; make correct decisions about class membership; and order numbers and objects by attributes.

Applying Knowledge and Understanding

Problem solving is a central aim, and often means, of teaching school mathematics, and hence this and supporting skills (e.g., select, represent, model) feature prominently in the *applying knowledge and conceptual understanding* domain. In items aligned with this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations and solve problems. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations are fundamental to success in the subject.

The problem settings are more routine than those aligned with the reasoning domain. The routine problems will typically have been

standard in classroom exercises designed to provide practice in particular methods or techniques. Some of these problems will have been in words that set the problem situation in a quasi-real context. Though they range in difficulty, each of these types of “textbook” problems is expected to be sufficiently familiar to students that they will essentially involve selecting and applying learned procedures.

Problems may be set in real-life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets. Therefore, problem solving is included not only in the *applying knowledge and conceptual understanding* domain, with emphasis on the more familiar and routine tasks, but also in the *reasoning* domain.

This cognitive domain covers the following behaviors:

Select	Select an efficient/appropriate operation, method or strategy for solving problems where there is a known algorithm or method of solution. Select simple algebraic expressions which represent straightforward situations (fourth grade). Select the nets of simple geometric figures (fourth grade). Select appropriate algorithms or formulas.
Represent	Display mathematical information and data in diagrams, tables, charts, or graphs, and generate equivalent representations for a given mathematical entity or relationship.
Model	Generate an appropriate model, such as an equation or diagram for solving a routine problem.
Implement	Follow and execute a set of mathematical instructions.

Solve Routine Problems	Solve routine problems (i.e., problems similar to those target students are likely to have encountered in class). For example, use geometric properties to solve problems; compare and match different representations of data (eighth grade) and use data from charts, tables, graphs, and maps to solve routine problems.
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Reasoning

Reasoning mathematically involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems. Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned. Non-routine problems may be purely mathematical or may have real-life settings. Both types of items involve transfer of knowledge and skills to new situations, and interactions among reasoning skills are usually a feature. Problems requiring reasoning may do so in different ways, because of the novelty of the context or the complexity of the situation or because any solution to the problem must involve several steps, perhaps drawing on knowledge and understanding from different areas of mathematics.

Even though of the other behaviors listed within the reasoning domain are those that may be drawn on in thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

This cognitive domain covers the following behaviors:

Analyze	Determine and describe or use relationships between variables or objects in mathematical situations; use proportional reasoning (fourth grade); decompose geometric figures to simplify solving a problem; draw the net of a given unfamiliar solid; visualize transformations of three-dimensional figures; compare and match different representations of the same data (fourth grade); and make valid inferences from given information.
Generalize	Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms.
Synthesize/ Integrate	Combine (various) mathematical procedures to establish results, and combine results to produce a further result. Make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas.
Justify	Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties.
Solve Non-Routine Problems	Solve problems set in mathematical or real life contexts where target students are unlikely to have encountered closely similar items, and apply mathematical procedures in unfamiliar or complex contexts. Use geometric properties to solve non-routine problems.