CHAPTER 1

TIMSS Advanced 2015 Mathematics Framework

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The assessment framework for TIMSS Advanced—Mathematics is organized around two dimensions: a content dimension specifying the domains of subject matter to be assessed within mathematics (i.e., algebra, calculus, and geometry) and a cognitive dimension specifying the domains of thinking processes to be assessed (i.e., knowing, applying, and reasoning). The cognitive domains describe the sets of behaviors expected of students as they engage with the mathematics content.

In general, these frameworks are similar to those used in TIMSS Advanced 2008. However, there have been minor updates to particular topics to better reflect the curricula, standards, and frameworks of the participating TIMSS Advanced countries. Also, attention was paid to current research and initiatives concerning mathematics and mathematics education, such as the *Common Core State Standards for Mathematics* (National Governors Association, 2010) developed in the United States, the *Mathematics Higher 2 Syllabus* (Singapore Examinations and Assessment Board, 2013) used in Singapore, the *Mathematics Curriculum (Secondary 4–6)* (Education Bureau, Hong Kong SAR, 2007) used in Hong Kong, and the *AP Calculus Course Description* (College Board, 2012).

Exhibit 1 shows the target percentages of testing time devoted to each content and cognitive domain for the advanced mathematics assessment.

Exhibit 1: Target Percentages of the TIMSS Advanced 2015 Mathematics Assessment Devoted to Content and Cognitive Domains

Content Domains	Percentages
Algebra	35%
Calculus	35%
Geometry	30%

Cognitive Domains	Percentages
Knowing	35%
Applying	35%
Reasoning	30%

TIMSS Advanced—Mathematics Content Domains

The TIMSS Advanced—Mathematics Framework consists of three content domains: algebra, calculus, and geometry. These content domains are the same content domains as were in the TIMSS Advanced 2008 Framework. Each of these content domains consists of topic areas, and each topic area in turn includes several topics. Across the advanced mathematics assessment, each topic receives approximately equal weight in terms of time allocated to assessing the topic.

Algebra

Algebra provides a foundation for further studies in mathematics as well as in many other disciplines. Building on the knowledge and skills developed in lower grades, the algebra domain encompasses three topic areas:

- Expressions and operations;
- Equations and inequalities; and
- Functions.

The first area includes operating with and evaluating a variety of algebraic expressions as well as working with arithmetic and geometric series. The second area includes using equations and inequalities, and systems of equations and inequalities to solve problems. The third area focuses on various representations and properties of functions.

Algebra: Expressions and Operations

1. Operate with exponential, logarithmic, polynomial, rational, and radical expressions; and perform operations with complex numbers.
2. Evaluate algebraic expressions (e.g., exponential, logarithmic, polynomial, rational, and radical).
3. Determine the n^{th} term of arithmetic and geometric series and the sums of finite and infinite series.

Algebra: Equations and Inequalities

1. Solve linear and quadratic equations and inequalities as well as systems of linear equations and inequalities.
2. Solve exponential, logarithmic, polynomial, rational, and radical equations.
3. Use equations and inequalities to solve contextual problems.

Algebra: Functions

1. Interpret, relate, and generate equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words.
2. Identify and contrast distinguishing properties of exponential, logarithmic, polynomial, rational, and radical functions.

Calculus

Calculus is an essential tool for understanding the principles governing the physical world and is the principal point of entry to most mathematically-based scientific careers. The calculus content for TIMSS Advanced—Mathematics concentrates on the following:

- Limits;
- Derivatives; and
- Integrals.

The focus is on understanding limits and finding the limit of a function, differentiation, and integration of a range of functions, and using these skills in solving problems.

Calculus: Limits

1. Determine limits of functions, including rational functions.
2. Recognize and describe the conditions for continuity and differentiability of functions.

Calculus: Derivatives

1. Differentiate polynomial, exponential, logarithmic, trigonometric, rational, radical, and composite functions; and differentiate products and quotients of functions.
2. Use derivatives to solve problems in optimization and rates of change.
3. Use first and second derivatives to determine slope, extrema, and points of inflection of polynomial and rational functions.
4. Use first and second derivatives to sketch and interpret graphs of functions.

Calculus: Integrals

1. Integrate polynomial, exponential, trigonometric, and simple rational functions.
2. Evaluate definite integrals, and apply integration to compute areas and volumes.

Geometry

Applications of geometry are tied directly to the solution of many real-world problems and are used extensively in the sciences. Because trigonometry has its origins in the study of triangle measurement, the geometry content domain also includes elements of trigonometry. The TIMSS Advanced 2015 geometry domain focuses on two topic areas common to most participating countries' curricula:

- Non-coordinate and coordinate geometry; and
- Trigonometry.

The focus of non-coordinate and coordinate geometry is on using the properties of geometric figures to solve problems in two and three dimensions, solving problems with coordinate geometry in two dimensions, and vectors. The other topic area concentrates on triangle trigonometry and trigonometric functions.

Geometry: Non-coordinate and Coordinate Geometry

1. Use non-coordinate geometry to solve problems in two and three dimensions.
2. Use coordinate geometry to solve problems in two dimensions.
3. Apply the properties of vectors and their sums and differences to solve problems.

Geometry: Trigonometry

1. Use trigonometry to solve problems involving triangles.
2. Recognize, interpret, and draw graphs of sine, cosine, and tangent functions.
3. Solve problems involving trigonometric functions.

TIMSS Advanced—Mathematics Cognitive Domains

The mathematics cognitive dimension consists of three domains based on what thinking processes students are expected to use when confronting the mathematics items developed for the TIMSS Advanced 2015 assessment. The first domain, knowing, addresses the students' ability to recall and recognize facts, procedures, and concepts necessary for a solid foundation in mathematics. The second domain, applying, focuses on using this knowledge to model and implement strategies to solve problems. The third domain, reasoning, includes analyzing, synthesizing, generalizing, and justifying through mathematical arguments or proofs. The situations requiring reasoning often are unfamiliar or complex.

While there is some hierarchy across the three cognitive domains (from knowing to applying to reasoning), each domain contains items representing a full range of difficulty. The following sections further describe the thinking skills and behaviors defining the cognitive domains. The general descriptions are followed by lists of specific behaviors to be elicited by items that are aligned with each domain.

Each content domain includes items developed to address each of the three cognitive domains. Accordingly, the algebra, calculus, and geometry domains include knowing, applying, and reasoning items.

Knowing

Knowing refers to students' knowledge of mathematical facts, concepts, and procedures. Mathematical facts and procedures form the foundation for mathematical thought.

Recall	Recall definitions, terminology, notation, mathematical conventions, number properties, and geometric properties.
Recognize	Recognize entities that are mathematically equivalent (e.g., different representations of the same function).
Compute	Carry out algorithmic procedures (e.g., determining derivatives of polynomial functions, and solving a simple equation).
Retrieve	Retrieve information from graphs, tables, texts, or other sources.

Applying

The applying domain involves the application of mathematics in a range of contexts. In this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations and solve problems. The problems in this domain typically reflect standard types of problems expected to be familiar to students. Problems may be set in real-life situations, or may be purely mathematical in nature involving, for example, numeric or algebraic expressions, functions, equations, or geometric figures.

Determine	Determine efficient and appropriate methods, strategies, or tools for solving problems for which there are commonly used methods of solution.
Represent/Model	Generate an equation or diagram that models problem situations and generate equivalent representations for a given mathematical entity, or set of information.
Implement	Implement strategies and operations to solve problems in familiar mathematical concepts and procedures.

Reasoning

Reasoning mathematically involves logical, systematic thinking. Problems requiring reasoning may do so in different ways, because of the novelty of the context or the complexity of the situation, the number of decisions and

steps, and may draw on knowledge and understanding from different areas of mathematics. Reasoning involves formulating conjectures, making logical deductions based on specific assumptions and rules, and justifying results.

Analyze	Identify the elements of a problem and determine the information, procedures, and strategies necessary to solve the problem.
Integrate/Synthesize	Link different elements of knowledge, related representations, and procedures to solve problems.
Evaluate	Determine the appropriateness of alternative strategies and solutions.
Draw Conclusions	Make valid inferences on the basis of information and evidence.
Generalize	Make statements that represent relationships in more general and more widely applicable terms.
Justify	Provide mathematical arguments, or proofs to support a strategy, solution, or statement.